

# A Mechanism Design Approach for Energy Allocation

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**Abstract.** In this work we deploy a mechanism design approach for allocating a divisible commodity (electricity in our example) among consumers. We consider to have an energy availability function and for each consumer an associated personal valuation function of the energy resource during a certain time interval. We aim to select the optimal consumption profile for every user avoiding consumption peaks when the total required energy could exceed the energy production. Initially, we apply a Vickrey-Clarke-Groves (VCG) mechanism, we show its properties and we discuss its weakness. Then, we describe our future works, developing a mechanism with verification. Our aim is to guarantee the maximization of social welfare (efficiency) and satisfy the budget balance property (the distributor's income must remain stable). So, we evaluate to define a cost-sharing mechanism in which for each tick of time if we exceed the amount of available energy the cost of exceeding energy provided by the distributor will be shared among users, deploying the Shapley value.

**Keywords:** mechanism design, resource allocation, energy efficiency.

## 1 Introduction

In this work, we face a problem that involves a community of users and a set of resources: the energy. The issue deals with the management of users' consumption according to the amount of available energy. There are several reasons for managing energy, first of all avoiding blackouts. A blackout could occur in situations in which the produced energy is not enough to satisfy users' energy demand and it causes a huge loss of money and security risks of the electricity-based systems [4]. Moreover, in a renewable world, energy production change day by day consequently we have to adequate energy requests. The European Commission itself finances projects in which users are stimulated to behave in an energy-aware manner according to energy saving targets (20% cut in greenhouse gas emissions, Targets 2020 [3]). Our main objective is to select the users' consumption amount in order to optimize and not to waste the produced energy. So, we are facing an energy management problem, also know as demand side

management (DSM) problem or also divisible commodities allocation[5]. Fig. 1 describes a possible case for one-day time period with trends for the energy functions. The dashed line represents the distributor's available energy, the continuous line the energy requested from all consumers. The lines on the bottom represent the single consumption of every user (five users in this case).

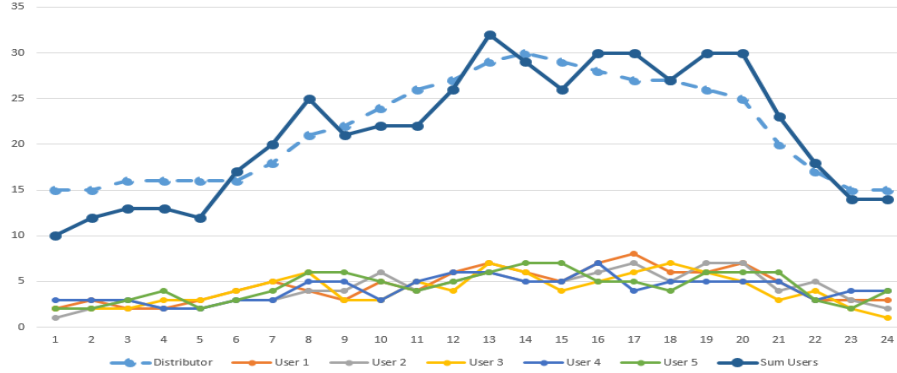


Fig. 1: Comparison between energy functions: users' desired trends, the aggregated desired trend of all of agent and the produced energy function.

## 2 Vickrey-Clarke-Groves (VCG) Mechanisms

We selected the subject of Mechanism Design [10,12,16,8], because it is able to influence users' actions by defining a game. This game is essentially composed by a user's valuation function (own utility), a social choice function (social welfare) and a payment scheme.

**Definition 1 (Player's Valuation Function)** *Let us consider a set of players  $N = \{1, \dots, n\}$  and a set of alternatives or outcomes  $A$ . Every player  $i$  has a preference over alternatives that is described by a valuation function:  $v_i : A \rightarrow \mathbb{R}$ , where  $v_i(a)$  denotes the valuation that player  $i$  assigns to outcome  $a$ . Furthermore,  $v_i \in V_i$  where  $V_i \subseteq \mathbb{R}^{|A|}$  is a set of possible valuation functions for player  $i$ .*

**Definition 2 (Social Choice Function)** *The social choice function selects an alternative (or outcome) from the set of alternatives  $A$  according to the vector of users' valuation functions:  $f : V_1 \times \dots \times V_n \rightarrow A$ . So, an outcome  $a$ , from the set  $A$  of alternatives, depends on each possible profile  $v = (v_1, v_2, \dots, v_n)$ :  $a = f(v)$ . This outcome is called social choice for that profile.*

When considering a mechanism with money, that is a mechanism where there are money transfers between the mechanism and players, a payment function computes money transfers for every players.

**Definition 3 (VCG Mechanism with Clarke Pivot Rule)** A VCG mechanism determines  $f(v)$  that is the social choice function with  $A$  as the possible outcomes, such that:

$$f(v) \in \operatorname{argmax}_{b \in A} \sum_{j=1}^n v_j(b);$$

### 3 Model Description

An approach close to our work is [14], and its related previous work [15]. They encourage efficient energy consumption among users with a VCG mechanism. In our previous work [2,1], we deploy a mechanism design approach for allocating a divisible commodity (electricity in our example) among consumers. We aim to select the optimal consumption profile for every user avoiding consumption peaks driving users in shifting energy consumptions in different hours of the day.

**Case 1** In this mechanism we have a set of possible outcomes  $A \subseteq \{0, 1\}^n$  where the value  $a[i] = 0$  represents that the specific user  $i$  does not consume, the opposite for the value  $a[i] = 1$  if the user consumes. We consider the consumption of every consumer with a threshold value represented by the amount of produced energy. Indeed, we consider the available energy function as a parameter that influences the possible outcome. In fact, we do not consider all the possible elements of the vector  $A \subseteq \{0, 1\}^n$  but  $A = \{a : \sum_j^n x_j \cdot a[j] \leq P\}$ , where  $P$  is the energy available and  $x_i$  is the desired consumption of player  $i$ . In this case, every user has the valuation function  $v_i(a) = x_i \cdot a[i]$ .

Suppose that the total users' consumption is less or equal than produced energy, we have that the social choice function will select the outcome maximizing the sum of valuations.

**Case 2** In this second case, we propose a mechanism that assigns to users all the available energy till reaching the total amount of produced energy.

Considering a scenario with  $n$  players, we have always a set of possible outcomes  $A \subseteq [0, x_i]^n$  where  $x_i$  is the desired consumption of user  $i$ . As in the case in Section 3, we do not consider all the possible elements of  $A$  but  $A = \{a : \sum_j^n a[j] \leq P\}$  and  $P$  is the energy available. In this way, the distributor can provide an arbitrary amount of energy to user  $i$  where the maximum  $x_i$  is the  $i$ 's optimal consumption. The valuation function of user  $i$  is:  $v_i(a) = a[i]$ .

**Case 3** We start from the mechanism described in Section 3. But, the main difference is that in this fourth case we take into account also the time  $t$ . Formally, we introduce a time variable:  $t \in [0, T]$ , where  $T$  is the maximum length.

Considering a scenario with  $n$  players, we have that a possible outcome is  $a = (a_1(t), \dots, a_n(t))$  where  $a_i(t) \in \mathbb{R}^+$  for every  $t \in [0, T]$  and where  $x_i : [0, T] \rightarrow \mathbb{R}$  is the desired consumption function of user  $i$  over time  $t$ . The function  $a_i(t)$  represents the assigned power to user  $i$  for every tick  $t$ , in the same way the function  $P(t)$  is the function of available power for every tick  $t$ , where the energy is the power per unit time.

The valuation function is:

$$v_i(a) = \begin{cases} a_i(t) & \text{if } a_i(t) \leq x_i(t) \\ x_i(t) & \text{if } a_i(t) > x_i(t) \end{cases}$$

representing the total amount of energy received by the  $i$ -th customer once the excess energy has been discarded.

## 4 Ongoing and Future Work

In the last period, we applied a VCG mechanism for the DSM problem. We propose several configurations of the mechanism, starting from the simplest one to a more complicated configuration which has the property of assigning the available energy to users according to their desired energy minimizing the energy wasting while maximizing the aggregate utility of all users. A mechanism is composed of a social choice function and a payment scheme. In this case we use the VCG payment scheme but in this way users will not pay according to the amount of energy assigned to them but they pay a different amount with respect to their energy consumption. For this reason, we decided to study different solutions where the payment will be more related to the energy used.

Considering that this social choice process depends on the information collected from agents, they may find it convenient to misreport their preferences. For this reason, the VCG mechanism is used in a context in which the “truthfulness” is a basic aspect, in fact the dominant strategy for a user is to declare his real preferences. On our specific case, we do not have to deal with truth or lies but the concept of truth is represented by the concept of “consume according to the available energy” and the lie by the concept of “consume when there is no available energy”. So, the punishment comes when a user consume too much and the consumption is greater than the available energy. Thus, we want to develop a mechanism with verification. In fact, in this context of fair allocations of goods with monetary compensation is possible to focus on agents’ declarations on allocated goods that can be verified before payments are performed. But, the verifier could evaluate only what it has already happened. First of all we allocate the goods then the mechanism control if users have behaved as they had said. Then, we deploy the Compensation and Bonus Mechanism [11] that concerns an optimal allocation algorithm with a payment function in which the payment function is sum of two terms: compensation and bonus. The compensation function is a monetary compensation considering actual types verified. While, the bonus function is calculated according to the declarations of the other agents and the actual times that the agent performed its assignments in. Furthermore, it is proved that the Compensation and Bonus mechanism is strongly truthful [11]. So, in this mechanism punishments are used to enforce truthfulness but in this case fairness is not guaranteed.

Our aim is to guide users to a energy-aware behaviour not to encourage users to tell the truth. Furthermore, we must define a fair allocation of resources with a different payment scheme that makes it convenient to consume where the energy is sufficient. The final objectives of our mechanism will first of all to guarantees

the maximization of social welfare (efficiency) and satisfies the budget balance property (the distributor's income must remain stable).

These characteristics could be reached by the definition of a cost-sharing mechanism in which for each tick of time if we exceed the amount of available energy the cost of exceeding energy provided by the distributor will be shared among users. This cost division could be calculated by the Shapley value [17]. The Shapley value includes several properties. The most important ones are fairness, ensuring that every user gets or is not damaged more than another one, and budget-balance, guaranteeing that there is no transfer of money out or into the scenario or in other words, mechanisms cannot run into deficit.

The mechanism will be subsequently tested by a multi-agent system, this framework will take into account also users' types (riches and poors, waster and environmentalist,...) starting from a set of real consumption data.

After the simulation phase, we will evaluate the existence of different kind of equilibria. It was showed that determining whether a game has a pure Nash Equilibrium is NP-hard [7]. However, it is still possible to bring hypothesis on the existence of different equilibria. As final step, we can work to minimize resulting costs for the energy with a user tax method that will influence users in acting in a convenient way considering the available energy function.

An important aspect to take into account is that all users involved are not of the same type or have identical preferences. In fact, in real-world scenarios we have to deal with heterogeneous agents that have different perception of energy efficiency, different lifestyles and different economic possibilities. In [6] there is a significant study of patterns of domestic electricity consumption. It evaluates a direct correlation between the amount of energy consumed and the user's well-being. Furthermore in [13], authors deploy evolutionary computing methods for auction designs extended by using heterogeneous trading agents. In this work, they review the method of using genetic algorithms for designing market mechanism with two different kind of artificial agents.

Until now, we assume to deal with fully rational agents but sometimes agents decide their strategy according to their moral or ethical beliefs. Considering this concept, the mechanism becomes quite difficult regarding the incentive compatibility. In the literature, we can find several papers that considering this aspect explain the concept of robustness of an incentive mechanism. As in [9], authors evaluate the robustness, defined as the maximum percentage of irrational agents existing in the system while it is still better off for rational agents to perform desired strategies. They provide a simulation framework for incentive mechanisms that calculates this robustness threshold.

So, the main difficulties are that we want to handle a community of heterogeneous users and according to their belief also irrational. Irrational in the sense that for instance a rich user is not interested in saving a small amount of money, he prefers to have a slightly greater level of comfort.

## References

1. S. Bistarelli, R. Culmone, P. Giuliadori, and S. Mugnoz. Mechanism Design Approach for Energy Efficiency. *ArXiv e-prints*, Aug. 2016.
2. S. Bistarelli, R. Culmone, P. Giuliadori, and S. Mugnoz. Mechanism design for allocation of divisible goods. In *Proceedings of 17th Italian Conference on Theoretical Computer Science (ICTCS 2016)*. CEUR Workshop Proceedings, 2016. to appear.
3. C. Böhringer, T. F. Rutherford, and R. S. Tol. The eu 20/20/2020 targets: An overview of the emf22 assessment. ESRI working paper 325, Dublin, 2009.
4. M. Bruch, V. Mnch, M. Aichinger, M. Kuhn, M. Weymann, and G. Schmid. Power Blackout Risks. Technical report, Allianz, 11 2011.
5. C. Gellings. The concept of demand-side management for electric utilities. *Proceedings of the IEEE*, 73(10):1468–1470, Oct. 1985. ISSN:0018-9219, DOI:10.1109/PROC.1985.13318.
6. S. Ghaemi and G. Brauner. User behavior and patterns of electricity use for energy saving. *Internationale Energiewirtschaftstagung an der TU Wien, IEWT*, 2009.
7. G. Gottlob, G. Greco, and F. Scarcello. Pure nash equilibria: Hard and easy games. *Journal of Artificial Intelligence Research (JAIR)*, 24:357–406, 2005.
8. M. O. Jackson. Mechanism theory. In U. Derigs, editor, *EOLSS The Encyclopedia of Life Support Systems*. EOLSS Publishers: Oxford UK, 2003.
9. Y. Liu and J. Zhang. Robustness evaluation of incentive mechanisms. In *Proceedings of the 2013 International Conference on Autonomous Agents and Multi-agent Systems, AAMAS '13*, pages 1293–1294, Richland, SC, 2013. International Foundation for Autonomous Agents and Multiagent Systems.
10. Y. Narahari. *Game Theory and Mechanism Design*, chapter 14. World Scientific Publishing Company Pte. Limited, 2014.
11. N. Nisan and A. Ronen. Algorithmic mechanism design (extended abstract). In *Proceedings of the Thirty-first Annual ACM Symposium on Theory of Computing, STOC '99*, pages 129–140, New York, NY, USA, 1999. ACM.
12. N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani. *Algorithmic Game Theory*, chapter 9. Cambridge University Press, 2007.
13. Z. Qin. Market mechanism designs with heterogeneous trading agents. *2006 International Conference on Machine Learning and Applications*, pages 69–76, 2006.
14. P. Samadi, H. Mohsenian-Rad, R. Schober, and V. W. S. Wong. Advanced demand side management for the future smart grid using mechanism design. *IEEE Transactions on Smart Grid*, 3(3):1170–1180, Sept 2012.
15. P. Samadi, R. Schober, and V. W. S. Wong. Optimal energy consumption scheduling using mechanism design for the future smart grid. In *SmartGridComm*, pages 369–374. IEEE, 2011.
16. Y. Shoham and K. Leyton-Brown. *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*, chapter 10. Cambridge University Press, New York, NY, USA, 2008.
17. E. Winter. The shapley value. In R. Aumann and S. Hart, editors, *Handbook of Game Theory with Economic Applications*, volume 3, chapter 53, pages 2025–2054. Elsevier, 1 edition, 2002.